Ordinary Differential Equations

Simply put, a differential equation is an equation expressing a relationship between a function and one or more of its derivatives. A function that satisfies a differential equation is called a *solution*.

The *Mathematica* command **DSolve** is used to solve differential equations. As with algebraic or transcendental equations, a double equal sign, **= =**, is used to separate the two sides of the equation.

**DSolve[*equation*, y[x], x]** gives the general solution, y[x], of the differential equation, *equation*, whose independent variable is x.

**DSolve[*equation*, y, x]** gives the general solution, y, of the differential equation expressed as a “pure” function within a list. **ReplaceAll** (**/.**) may then be used to evaluate the solution. Alternatively, one may use **Part** or **[[ ]]** to extract the solution from the list.

**Ex.1** Solve the first-order differential equation

**Ex.2** Solve the equation with initial condition *y*(0) = 2.Then plot the solution.

A useful way of visualizing the solution of a first-order differential equation is to introduce the concept of a vector field. A vector field on is a vector function that assigns to each point (*x*, *y*) a two-dimensional vector **F**(*x*, *y*). By drawing the vectors **F**(*x*, *y*) for a (finite) subset of , one obtains a geometric interpretation of the behavior of **F**.

Ex.3 Plot the vector field **F**(*x*, *y*) = – *y* **i** + *x* **j** .

**Any first-order differential equation can be used to define a vector field. Indeed, the vector field i + *f*(*x*, *y*) j, corresponding to the equation**

**, generates a field whose vectors are tangent to the solution. The next example, although simple, illustrates this nicely.**

**Ex.4** Plot the vector field of the solution of the equation . The solutions to this equation, parabolas , can be seen easily.

**Ex.5** Plot the vector field generated by the equation *.* Next plot the solutions with initial conditions *y*(0) = –2, –1, 0, 1, and 2 b for comparison with the plot with the vector field.

**Ex.6**

Plot the vector field for the equation together with its solutions for *y*(0) = 0, 1, 2, 3, and 4.

**Ex.7**

to Newton’s law of cooling, the temperature of an object changes at a rate proportional to the difference in temperature between the according object and the outside medium. If an object whose temperature is 70°F is placed in a medium whose temperature is 20°, and is found to be 40° after 3 minutes, what will its temperature be after 6 minutes?

(hint: If *u*(*t*) represents the temperature of the object at time *t*,.

**Ex.8**

The equation governing the amount of current, *I*, flowing through a simple resistance–inductance circuit when an EMF (voltage) *E* is applied is . The units for *E*, *I*, and *L* are, respectively, volts, amperes, and henries. If *R* = 10 ohms, *L* = 1 henry, the EMF source is an alternating voltage whose equation is *E*(*t*) = 10 sin 5*t*, and the current is initially 4 amperes, find an expression for the current at time *t* and plot the graph of the current for the first 3 seconds.

**Ex.9**

If a spring with mass *m* attached at one end is suspended from its other end, it will come to rest in an equilibrium position. If the system is then perturbed by releasing the mass with an initial velocity of at a distance *y*0 below its equilibrium position, its motion satisfies the differential equation , , , *a* is a damping constant (determined experimentally) due to friction and air resistance, and *k* is the spring constant given in Hooke’s law.

A mass of Ľ slug is attached to a spring with a spring constant, *k*, of 6 lb/ft. The mass is pulled downward from its equilibrium position 1 ft and then released. Assuming a damping constant, *a*, of 1/2, determine the motion of the mass and sketch its graph for the first 5 seconds.

**Ex.10**

Consider the differential equation with initial condition *y*(0) = 1.

Although this equation has a unique solution, it cannot be found in terms of elementary functions using **DSolve**. Find the numerical solution